IMSc-97/11; hep-th yy/mm/nnn

Black Hole Radiance and Supersymmetry

Parthasarathi Majumdar*

The Institute of Mathematical Sciences, CIT Campus, Madras 600113, India.

Abstract

Starting with free massless scalar and spinor fields described by a globally $N = 1$ supersymmetric action, infalling on a Schwarzschild black hole, the outgoing Hawking radiation is shown to break supersymmetry spontaneously, exactly as induced by a heat bath in Minkowski space, with no generation of Nambu-Goldstone fermions.

Bose-fermi asymmetry appears to be intrinsic to the formulation of singularity theorems in general relativity [1]. The weak energy condition is certainly not valid for spinor fields, while for classical bosonic particles it is perfectly consistent with phenomena like the Penrose process [2]. Perhaps for related reasons, this dissimilarity persists through quantization of the respective matter fields in classical black hole spacetimes. Indeed, superradiance of spin-1/2 particles is severely suppressed, in contrast to integral spin fields of appropriate charge/angular momentum [3]. Furthermore, in Hawking's celebrated theory of black hole evaporation [4], integral spin fields are radiated in a Planckian (Bose-Einstein) spectrum at a 'temperature' proportional to the surface gravity of the black hole. Spinor fields, on the other hand, are radiated in accord with Fermi-Dirac statistics at the same temperature. Of course, one can scarcely overemphasize that this 'thermal' nature of the Hawking spectrum has *no* obvious statistical mechanical underpinning, in contrast to standard thermodynamics; it is

*email: partha@imsc.ernet.in

arrived at by analogy, albeit a compelling one. Until satisfactory micro-foundations for this are discerned, signals of thermal behaviour in semi-classical black hole physics ought, in our opinion, to be carefully analyzed on a case-by-case basis. This spirit underlies the present investigation.

We focus on a situation where the infalling matter is (globally) supersymmetric to begin with; in particular, at past null infinity ($\mathcal{I}^-$), we envisage a globally supersymmetric model of noninteracting massless complex scalar and chiral spinor fields. The Hilbert space of the theory has a unique supersymmetric vacuum with particle-excitations forming irreducible representations of the supersymmetry algebra. Now, any state on $\mathcal{I}^-$ will evolve into a state on the event horizon ($\mathcal{H}^+$), belonging to one of two mutually exclusive (in absence of backreaction) classes, viz., those which are purely outgoing, i.e., have zero Cauchy data on $\mathcal{H}^+$ and support on $\mathcal{I}^+$, and those which have zero Cauchy data on $\mathcal{I}^+$ and support on $\mathcal{H}^+$. As is well-known [4], an inherent ambiguity in the latter is chiefly responsible for the thermalization of the radiation received at $\mathcal{I}^+$. It is obvious that this randomness emasculates the supermultiplet structure one had at $\mathcal{I}^-$. Our concern here is with the nature of the violence, especially in comparison with supersymmetry breakdown induced by finite temperature effects in *Minkowski* space [5]. Once again, there is no a-priori reason for these two phenomena to be identical, but they turn out to be so, in unexpected detail.

The scalar and spinor fields in our model have the following expansion (at $\mathcal{I}^-$),

$$\begin{aligned} \phi &= \sum_k \frac{1}{\sqrt{2\omega_k}} \left(a_k^B f_k^B + b_k^{B\,\dagger} \bar{f}_k^B \right) \\ \psi_+ &= \sum_k \frac{1}{\sqrt{2\omega_k}} \left(a_{k,+}^F f_k^F + b_{k,-}^{F\,\dagger} \bar{f}_k^F \right) u_{k,+} \;, \end{aligned} \tag{1}$$

where, the $\{f_k\}$ are complete orthonormal sets of solutions of the respective field equations, with positive frequencies only at $\mathcal{I}^-$, and $u_{k,+}$ is a positively chiral spinor, reflecting the chirality of ψ_+. The creation-annihilation operators obey the usual algebra, with B (F) signifying Bose (Fermi). The conserved Nöther supersymmetry charge is given in terms of these creation-annihilation operators by (at $\mathcal{I}^-$)

$$Q_+(\mathcal{I}^-) \;=\; \sum_k \left(a^F_{k,+} b^{B\dagger}_k \;-\; b^F_{k,-}{}^\dagger a^B_k \right) u_+(k) \;, \tag{2}$$

and annihilates the vacuum state $|0_- >$ defined by

$$a^{B,F}_k |0_- > \;=\; 0 \;=\; b^{B,F}_k |0_- > \quad . \tag{3}$$

The existence of two disjoint classes of states at the horizon, as mentioned earlier, imply that the fields also admit the expansion [4]

$$\begin{aligned} \phi \;&=\; \sum_k \frac{1}{\sqrt{2\omega_k}} \left(A^B_k p^B_k \;+\; B^B_k{}^\dagger \bar{p}^B_k \;+\; A'^B_k q^B_k \;+\; B'^B_k{}^\dagger \bar{q}^B_k \right) \\ \psi_+ \;&=\; \sum_k \frac{1}{\sqrt{\omega_k}} \left(A^F_{k,+} p^F_k \;+\; B^F_{k,+}{}^\dagger \bar{p}^B_k \;+\; A'^F_{k,+} q^F_k \;+\; B'^F_{k,-}{}^\dagger \bar{q}^F_k \right) u_+(k) \quad , \end{aligned} \tag{4}$$

where, $\{p_k\}$ are purely outgoing orthonormal sets of solutions of the respective field equations with positive frequencies at $\mathcal{I}^+$, while $\{q_k\}$ are orthonormal sets of solutions with no outgoing component. The final vacuum state $|0_+ >$, defined by the requirement

$$A^{B,F}|0_+ > \;=\; 0 \;= B^{B,F}|0_+ > \;=\; A'^{B,F}|0_+ > \;=\; B'^{B,F}|0_+ > \tag{5}$$

is not unique, because of the inherent ambiguity in defining positive frequency for the $\{q_k\}$; in fact, one can write $|0_+ >= |0_I > |0_H >$ with the unprimed (primed) operators acting on $|0_I >$ ($|0_H >$). The ambiguity in $|0_H >$ results in particle creation at the horizon with eventual radiation out to future infinity as a thermal spectrum of positive energy particles. Note, however, that a supersymmetry charge $Q(\mathcal{I}^+)$ may indeed be defined, analogously to eqn. (2), in terms of the unprimed operators, and that $Q(\mathcal{I}^+)|0_+ > \;=\; 0$. Such a charge also satisfies the $N = 1$ superalgebra at $\mathcal{I}^+$.

The field operators a_k, b_k at $\mathcal{I}^-$ are of course related to the A_k, B_k and A'_k, B'_k through the Bogoliubov transformations

$$\begin{aligned} A^B_k \;&=\; \sum_{k'} \left(\alpha^B_{kk'} a^B_{k'} \;+\; \beta^B_{kk'} b^B_{k'}{}^\dagger \right) \\ B^B_k \;&=\; \sum_{k'} \left(\alpha^B_{kk'} b^B_{k'} \;+\; \beta^B_{kk'} a^B_{k'}{}^\dagger \right) \\ A^F_{k,+} \;&=\; \sum_{k'} \left(\alpha^F_{kk'} a^F_{k',+} \;+\; \beta^F_{kk'} b^F_{k',-}{}^\dagger \right) \\ B^F_{k',-} \;&=\; \sum_{k'} \left(\alpha^F_{kk'} b^F_{k',-} \;+\; \beta^F_{kk'} a^F_{k',+}{}^\dagger \right) \;, \end{aligned} \tag{6}$$

and similarly for the primed operators. We notice in passing that

$$Q(\mathcal{I}^-)|0_+ > \;\neq\; 0\;,\; Q(\mathcal{I}^+)|0_- > \;\neq\; 0\;,\; for\; \beta^{B,F} \;\neq\; 0\;. \tag{7}$$

The issue the we now wish to focus on is whether the radiated system of particles has $N = 1$ spacetime supersymmetry. To address this question, recall that vacuum expectation values (vevs) of observables at future null infinity are defined by [4]

$$< \mathcal{O} > \;\equiv\; < 0_-|\mathcal{O}|0_- > \;=\; Tr(\rho\mathcal{O}) \tag{8}$$

where, ρ is the density operator. The trace essentially averages over the (nonunique) states going through the horizon, thus rendering the vevs of observables (at $\mathcal{I}^+$) free of ambiguities. We also recall that a sufficient condition for spontaneous supersymmetry breaking is the existence of *a* fermionic operator $\mathcal{O}$ which, upon a supertransformation, yields an operator with non-vanishing vev, i.e., $< \delta_S\mathcal{O} > \;\neq\; 0$. Thus, if one is able to show that for *all* fermionic observables $\mathcal{O}(\mathcal{I}^+)$,

$$< 0_-|\delta_S\mathcal{O}(\mathcal{I}^+)|0_- > \;=\; 0\;, \tag{9}$$

then we are guaranteed that the outgoing particles form a supermultiplet.

However, this is not the case, as is not difficult to see; for, consider the supercharge operator itself at $\mathcal{I}^+$. Using the supersymmetry algebra, it can be shown that

$$< 0_-|\delta_S Q(\mathcal{I}^+)|0_- > \;=\; \bar{\epsilon}\gamma_\mu \;< 0_-|P^\mu(\mathcal{I}^+)|0_- >\;, \tag{10}$$

where P^μ is the momentum operator of the theory. In our free field theory, the rhs of (10) is trivial to calculate, using eq. (6) above, so that we obtain

$$\begin{aligned} <\; \delta_S Q(\mathcal{I}^+)\; > \;&= \bar{\epsilon}\gamma \;\cdot \sum_k k <\; N_k^B \;+\; N_k^F \;> \\ &= \bar{\epsilon}\gamma \;\cdot\; \sum_{k,k'} k \left(\, |\beta_{kk'}^B|^2 \;+\; |\beta_{kk'}^F|^2 \,\right)\;. \end{aligned} \tag{11}$$

Thus, supersymmetry is spontaneously broken in the sense described above, so long as the

Bogoliubov coefficients $\beta^{B,F}$ are non-vanishing[1]. In fact, we know from Hawking's seminal work [4] that

$$\begin{aligned} < N_k^B > &= \sum_{k'} |\beta_{kk'}^B|^2 = |t_{|k|}|^2 \left(e^{2\pi|k|/\kappa} - 1 \right)^{-1} \\ < N_k^F > &= \sum_{k'} |\beta_{kk'}^F|^2 = |t_{|k|}|^2 \left(e^{2\pi|k|/\kappa} + 1 \right)^{-1} . \end{aligned} \tag{12}$$

Here, κ is the surface gravity of the Schwarzschild black hole and given by $M/4$ where M is the black hole mass.

It is instructive to compare these results, in particular, those expressed by eq.s (11) and (12), with a flat space calculation of the thermal ensemble average of $\delta_S Q$, assuming an ideal gas of massless scalar and chiral spinor particles in equilibrium with a heat bath at a temperature T. Following [5], finite temperature field theory can be used to show that, in our theory,

$$\begin{aligned} < \delta_S Q >_T &\equiv \frac{\int \mathcal{D}\phi \mathcal{D}\psi e^{-S/T} \delta_S Q}{\int \mathcal{D}\phi \mathcal{D}\psi e^{-S/T}} \\ &= \bar{\epsilon}\gamma \cdot \sum_k k \left[n_k^B(T) + n_k^F(T) \right] , \end{aligned} \tag{13}$$

where, $n_k^{B,F}(T) = (e^{\omega_k/T} \mp 1)^{-1}$. With the identification between the temperature T of the heat bath and the Hawking temperature $T_H \equiv \kappa/2\pi$, the similarity between supersymmetry breakdown in the two cases is unmistakable.

Next consider the possible generation of Nambu-Goldstone fermions; the standard proof of the Goldstone theorem has to be adapted to the black hole geometry. We shall ignore subtleties that this adaptation might elicit and make the following seemingly reasonable assumptions: (a) the generally covariant definition of the time ordered product in curved backgrounds, given e.g., in [2], can be written as

$$\mathcal{T}\ \mathcal{S}_\mu(x)\mathcal{O}(y) = \mathcal{S}_\mu \mathcal{O} \Theta(x,y) + \mathcal{O}\mathcal{S}_\mu \Theta(y,x) , \tag{14}$$

where,

---

[1] A hint of this was already available from eq. (7)

$$\Theta(x,y) = 1 \quad , when\ x \in J^+(y)$$
$$= 0 \quad otherwise\ ; \tag{15}$$

with $J^+(y)$ being the *causal future* of y; (b)the Θ function is also assumed to satisfy[2]

$$\mathcal{S}^\mu \nabla_{\mu,x}\Theta(x,y) = \mathcal{S}^0 \delta(x^0 - y^0) \tag{16}$$

(c) the orthonormal set of purely outgoing solutions $\{p_k(x)\}$ is to serve as the kernel for Fourier transformations, with the assumed properties

$$k_\mu p_k(x) = -i\nabla_\mu p_k(x)$$
$$\lim_{\vec{k}\to 0} p_k(\vec{x},0) = 1\ . \tag{17}$$

One might object to the apparent lack of manifest covariance in the above, but one hopes that the inferences to follow would be vindicated by a more rigorous treatment. Consider now the quantity

$$\lim_{\vec{k}\to 0} k^\mu \mathcal{F}_\mu(k) \equiv \lim_{\vec{k}\to 0} \bar{\epsilon} \int d^4x\sqrt{-g}\ k^\mu p_k(x) < 0_-|\mathcal{T}\mathcal{S}_\mu(x)\mathcal{O}_(y)|0_- > \ , \tag{18}$$

where, ϵ is the supersymmetry parameter, $\mathcal{S}_\mu$ is the Nöther supercurrent and $\mathcal{O}$ a local spinorial operator located at $y \in \mathcal{I}^+$. With the above assumptions, it is not difficult to show that

$$\lim_{\vec{k}\to 0} k^\mu \mathcal{F}_\mu = \bar{\epsilon} < 0_-|\delta_S \mathcal{O}(y)|0_- >$$
$$- i \lim_{\vec{k}\to 0} \int d^4x \nabla_\mu \left(p_k(x) < 0_-|\mathcal{T}\mathcal{S}^\mu(x)\mathcal{O}(y)|0_- >\right)\ . \tag{19}$$

The surface term in (19) is usually taken to vanish, so that, if $< \delta_S\mathcal{O}(0) > \neq 0$, a massless pole – the Nambu-Goldstone pole appears. However, in the situation under consideration, the surface term need not vanish, mainly because of eq. (7), although one does have $Q(\mathcal{I}^-)|0_- >= 0$. Thus, it is no longer true that the rhs of (19) is unambiguously nonzero, and hence the existence of the massless pole cannot be inferred.

---

[2]This is probably a valid assumption in a static metric like Schwarzschild

Once again, similar conclusions ensue from Minkowski space considerations at finite temperature: recall that conventionally, one Euclideanizes and compactifies the time coordinate and imposes suitable boundary conditions on the fields (viz., periodic or antiperiodic) [6] with respect to this coordinate. As can be easily shown [5], the surface terms do not vanish because of contributions from the boundary of the Euclideanized time coordinate. In our case, of course, no such compactification of coordinates is necessary, but similar non-vanishing of the surface terms results because of the nontriviality of the Bogoliubov coefficients β, and the identification of the surface gravity with the (Hawking) temperature.

The manner of supersymmetry breakdown in outgoing Hawking radiation, at least in the simple example dealt with here, resembles finite temperature supersymmetry breaking effects substantially. Indeed, from the perspective of the Euclidean functional integral approach [7] one may consider the foregoing results as obvious. That the same results emerge without appealing to the premises of such an approach is perhaps of some importance, especially in view of recent calculations [8] of the radiation formula, for some special cases, using the theory of Dirichlet branes [9] and their agreement [10] with results derived within the semiclassical analysis. The issue of supersymmetry breaking considered here will be important in those cases as well, more so because the Dirichlet branes under consideration are assumed to have some unbroken spacetime supersymmetry in most cases.

Extensions of the results to charged or rotating black holes may be accomplished by incorporation of a 'chemical potential': $\omega_k \rightarrow \omega_k - \mu \ where \ \mu = e\Phi \ or \ m\Omega$ [4]. It would be interesting to consider 'supersymmetric' black holes in supergravity theories, although one expects them to behave similarly as long as they emit Hawking radiation. Emission of photons and gravitons and their superpartners should not change the results qualitatively, so long as their mutual interactions can be neglected. One does not expect a breakdown of full nonlinear local supersymmetry in the conventional sense though; the absence of Nambu-Goldstone spinors should inhibit the super-Higgs effect, so that gravitini retain their masslessness. However, this issue clearly warrants a more thorough analysis.

Finally, a word on possible application of our results; clearly, Hawking radiation is phys-

ically irrelevant (i.e., from an observational standpoint) for large black holes because these typically would have very small surface gravity (Hawking temperature). The arena where this process becomes important is that of the early universe where density fluctuations may produce low mass black holes [4]. Now supersymmetry is known to play an important role in contemporary cosmological scenarios [11]. Vigourously evaporating black holes may produce interesting consequences in the inflationary epoch if our considerations regarding supersymmetry breakdown are valid. We hope to report on some of these issues elsewhere.

REFERENCES

[1] S. Hawking and G.Ellis, *The Large Scale Structure of Space-time*, pp 88-95 (Cambridge University Press, 1973).

[2] R. Wald, *General Relativity*, p. 399 *et. seq.* (University of Chicago, 1984).

[3] W. Unruh, Phys. Rev. **D10**, 3194 (1974).

[4] S. Hawking, Comm. Math. Phys. **43**, 199 (1975); Phys. Rev. **D14**, 2460 (1976).

[5] L. Girardello, M. Grisaru and P. Salomonson, Nucl. Phys. **B178**, 331 (1981)

[6] L. Dolan and R. Jackiw, Phys. Rev. **D9**, 3320 (1974).

[7] J. Hartle and S. Hawking, Phys. Rev. **D13**, 2188 (1976).

[8] C. Callan and J. Maldacena, Nucl. Phys. **B475**, 645 (1996).

[9] J. Polchinski, Phys. Rev. Lett. **75**, 4724 (1995).

[10] S. Das and S. Mathur, hep-th 9606185.

[11] A. Linde, *Particle Physics and Inflationary Cosmology* p 213 *et. seq.* (Harwood, 1990).